

165. If $x^2 - 4x + 1 = 0$ then find $(x^3 - 26)^2 = ?$

यदि $x^2 - 4x + 1 = 0$ तो $(x^3 - 26)^2$ ज्ञात कीजिये

[A] 545

[C] 625

☒ [B] 675

[D] 91.125

$$x + \frac{1}{x} = 4$$

$$x^3 + \frac{1}{x^3} = 52$$

$$\Rightarrow x^3(x^3 - 52) = -1$$

$$\underline{x^6 - 52x^3 + 676}$$

$$= x^3(x^3 - 52) + 676$$
$$= -1 + 676$$
$$= 675$$



166. If $x^2 + x = 5$, then $(x + 3)^3 + \frac{1}{(x+3)^3} = ?$

यदि $x^2 + x = 5$ है, तो $(x + 3)^3 + \frac{1}{(x+3)^3}$ का मान क्या है?

(RRB NTPC GRADUATE LEVEL CBT-1 2026)

✓ [A] 110
[C] 140

माना, $x+3=y$
 $\Rightarrow x=y-3$

[B] 125
[D] 0

$$y^3 + \frac{1}{y^3} = 5^3 - 3 \times 5 = 110$$

$$y^2 - 6y + 9 + y - 3 = 5$$

$$\frac{y^2 - 5y + 1 = 0}{y} \Rightarrow y + \frac{1}{y} = 5$$

167. If $a^2 + a = 13$, then find the value of $(a + 4)^3 - \frac{1}{(a+4)^3}$

यदि $a^2 + a = 13$, है तो $(a + 4)^3 - \frac{1}{(a+4)^3}$ का मान ज्ञात करे-

[A] 364 ज्ञाता $a+4=x$

[B] 540

[C] 440 $\Rightarrow a = x - 4$

[D] 420 $x^3 - \frac{1}{x^3} = 7^3 + 3 \times 7$
 $= 364$

$$x^2 - 8x + 16 + x - 4 = 13$$

$$\frac{x^2 - 7x - 1}{x} = 0$$

$$\Rightarrow x - \frac{1}{x} = 7$$

168. If $p^2 - 29p + 199 = 0$ find $(p - 11)^3 - \frac{1}{(p-11)^3} = ?$

यदि $p^2 - 29p + 199 = 0$ है, तो $(p - 11)^3 - \frac{1}{(p-11)^3}$ ज्ञात कीजिए।

[A] $135\sqrt{5}$ माना $p-11=x$

[B] $122\sqrt{5}$

[C] $144\sqrt{5}$

[D] $126\sqrt{5}$

$$x^3 - \frac{1}{x^3} = 48\sqrt{5}$$
$$= 48 \times 3\sqrt{5}$$
$$= 144\sqrt{5}$$

$$x^2 + \cancel{xx} + 11 - \cancel{xx} - 31x + 199 = 0$$

$$\frac{x^2 - 7x + 1}{x} = 0 \Rightarrow x + \frac{1}{x} = 7 \Rightarrow x - \frac{1}{x} = \sqrt{7^2 - 4} = \sqrt{45}$$

Algebra sheet-③

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$$\text{if, } x - \frac{1}{x} = 4$$

$$\text{तो } x^4 - \frac{1}{x^4} = (x^2)^2 - \left(\frac{1}{x^2}\right)^2 = \left(x^2 + \frac{1}{x^2}\right) \cdot \left(x + \frac{1}{x}\right) \cdot \left(x - \frac{1}{x}\right)$$

$$= 18 \times \sqrt{20} \times 4$$

$$= 72 \times 2\sqrt{5}$$

$$= 144\sqrt{5}$$

IF,

$$x + \frac{1}{x} = 6$$



$$x^2 + \frac{1}{x^2} = 34$$

$$x^4 + \frac{1}{x^4} = 34^2 - 2 = 1154$$

$$x^4 - \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right) \cdot \left(x + \frac{1}{x}\right) \cdot \left(x - \frac{1}{x}\right)$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ 34 & \times & 6 \\ & & \downarrow \\ & & \sqrt{32} \end{array}$$

$$= 204 \times 4\sqrt{2} = 816\sqrt{2}$$

CONCEPT



1. If $x^2 - \sqrt{8}x + 1 = 0$, then the value of $\left(x^4 - \frac{1}{x^4}\right)$ is:

यदि $x^2 - \sqrt{8}x + 1 = 0$ है, तो $\left(x^4 - \frac{1}{x^4}\right)$ का मान क्या होगा?

[A] $24\sqrt{6}$

[B] $24\sqrt{2}$

[C] $20\sqrt{2}$

[D] $20\sqrt{6}$

$$x + \frac{1}{x} = \sqrt{8}$$

$$\left(x^2 + \frac{1}{x^2}\right) \cdot \left(x + \frac{1}{x}\right) \cdot \left(x - \frac{1}{x}\right)$$

$$\downarrow \quad \times \quad \downarrow \quad \times \quad \downarrow$$

$$6 \quad \times \quad \sqrt{8} \quad \times \quad 2$$

$$= 12 \times 2\sqrt{2}$$

if, $x + \frac{1}{x} = k$ तो $x^5 + \frac{1}{x^5} = ?$

$$\left(x^2 + \frac{1}{x^2}\right) \cdot \left(x^3 + \frac{1}{x^3}\right) = x^5 + \frac{1}{x^5} + \underbrace{x + \frac{1}{x}}$$

$$\left(x^2 + \frac{1}{x^2}\right) \cdot \left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right) = x^5 + \frac{1}{x^5}$$

$$(x^2)^3 = x^6$$

$$x^2 \times x^3 = x^5$$

IF,

$$x + \frac{1}{x} = 5$$



CONCEPT



तो $x^5 + \frac{1}{x^5} = \left(x^2 + \frac{1}{x^2}\right) \cdot \left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)$

$\downarrow \quad \quad \downarrow$
 $23 \times 110 - 5$

$$= 2530 - 5$$

$$= 2525$$

2. If $\left(a + \frac{1}{a}\right) = 4$, then find the value of $\left(a^5 + \frac{1}{a^5}\right)$.
 यदि $\left(a + \frac{1}{a}\right) = 4$ है, तो $\left(a^5 + \frac{1}{a^5}\right)$ का मान ज्ञात कीजिए।

RRB NTPC Graduate Level CBT-1 2024

[A] 724

[C] 776

[B] 709

[D] 770

$$\left(a^2 + \frac{1}{a^2}\right) \cdot \left(a^3 + \frac{1}{a^3}\right) - \left(a + \frac{1}{a}\right)$$

$$\downarrow \quad \downarrow$$

$$14 \times 52 - 4$$

$$= 728 - 4$$

3. If $x^2 - 2\sqrt{5}x + 1 = 0$, then what is the value of $x^5 + \frac{1}{x^5}$?

यदि $x^2 - 2\sqrt{5}x + 1 = 0$ है तो $x^5 + \frac{1}{x^5}$ का मान क्या है?

[A] $408\sqrt{5}$

[B] $610\sqrt{5}$

[C] $406\sqrt{5}$

[D] $612\sqrt{5}$

$18 \times 34\sqrt{5} - 2\sqrt{5}$

$$x + \frac{1}{x} = 2\sqrt{5}$$

$$x^2 + \frac{1}{x^2} = 18$$

$$x^3 + \frac{1}{x^3} = (2\sqrt{5})^3 - 3 \times 2\sqrt{5} = 34\sqrt{5}$$

4. If $x > 0$, and $x^4 + \frac{1}{x^4} = 254$, what is the value of $x^5 + \frac{1}{x^5}$?

यदि $x > 0$, और $x^4 + \frac{1}{x^4} = 254$, तो $x^5 + \frac{1}{x^5}$ का मान क्या है?

[A] $717\sqrt{2}$

[B] $723\sqrt{2}$

[C] $720\sqrt{2}$

[D] $726\sqrt{2}$

$$\begin{aligned} & \rightarrow 16 \times 15\sqrt{18} - \sqrt{18} \\ & = 239\sqrt{18} \\ & = 239 \times 3\sqrt{2} \end{aligned}$$

$$x^2 + \frac{1}{x^2} = \sqrt{256} = 16$$

$$x + \frac{1}{x} = \sqrt{18}$$

$$x^3 + \frac{1}{x^3} = 15\sqrt{18}$$

5. If $X = 4 - \frac{1}{4 - \frac{1}{4 - \frac{1}{4 - \dots \infty}}}$, then find $X^5 + \frac{1}{X^5} = ?$

यदि $x = 4 - \frac{1}{4 - \frac{1}{4 - \frac{1}{4 - \dots \infty}}}$ है, तो $x^5 + \frac{1}{x^5} = ?$ ज्ञात कीजिए।

[A] 656

[C] 732

[B] 548

[D] 724

$$x = 4 - \frac{1}{x}$$

$$x + \frac{1}{x} = 4$$

$$x^5 - \frac{1}{x^5} = ?$$

$$\left(x^2 + \frac{1}{x^2}\right) \cdot \left(x^3 - \frac{1}{x^3}\right) = x^5 - \frac{1}{x} + x - \frac{1}{x^5}$$

$$\left(x^2 + \frac{1}{x^2}\right) \cdot \left(x^3 - \frac{1}{x^3}\right) - \left(x - \frac{1}{x}\right) = x^5 - \frac{1}{x^5}$$

IF, $x - \frac{1}{x} = 4$





तो $x^5 - \frac{1}{x^5} = \left(x^2 + \frac{1}{x^2} \right) \left(x^3 - \frac{1}{x^3} \right) - \left(x - \frac{1}{x} \right)$

$\downarrow$ $\downarrow$ $\downarrow$
 $4^2 + 2$ $4^3 + 3 \times 4$ 4

1260
 108

$= 18 \times 76 - 4$

$= 1368 - 4 = 1364 \checkmark$

6. If $y - \frac{1}{y} = -9$, then find $y^5 - \frac{1}{y^5}$?

यदि $y - \frac{1}{y} = -9$, तो $y^5 - \frac{1}{y^5}$ ज्ञात करें?

[A] ~~- 62757~~

[B] ~~- 62748~~

[C] ~~- 62739~~

[D] ~~- 59049~~

$83 \times 756 = 60000$

$$\begin{aligned} &(-9)^2 + 2 \\ &(-9)^3 + 3(-9) \end{aligned}$$

$$\begin{aligned} &\left(y^2 + \frac{1}{y^2}\right) \cdot \left(y^3 - \frac{1}{y^3}\right) - \left(y - \frac{1}{y}\right) \\ &\quad \downarrow \quad \quad \downarrow \\ &83 \times -756 + 9 \\ &= -(83 \times 756 - 9) \end{aligned}$$

IF,

$$x + \frac{1}{x} = 5$$



तो $x^6 + \frac{1}{x^6} = ?$

$$(x^2)^3 = (x^3)^2 = x^6$$

CONCEPT



$$x^3 + \frac{1}{x^3} = 5^3 - 3 \times 5 = 110$$

$$\begin{aligned} x^6 + \frac{1}{x^6} &= 110^2 - 2 \\ &= 12100 - 2 \\ &= 12098 \checkmark \end{aligned}$$

$$\text{if, } x - \frac{1}{x} = \sqrt{10}$$

$$x^3 - \frac{1}{x^3} = 13\sqrt{10}$$

$$\text{तो } x^6 + \frac{1}{x^6} = (13\sqrt{10})^2 + 2 \\ = 1692$$

7. $x^2 - 4x + 1 = 0$, then what is the value of $(x^6 + x^{-6})$?

यदि $x^2 - 4x + 1 = 0$ है, तो $(x^6 + x^{-6})$ का मान क्या है?

[A] 2786 ✗

[B] 2702 ✓

[C] 2716 ✗

[D] 2744 ✗

$$x + \frac{1}{x} = 4$$

$$x^3 + \frac{1}{x^3} = 4^3 - 3 \times 4$$

$$= 52$$

$$x^6 + \frac{1}{x^6} = 52^2 - 2$$

$$= 2702$$

8. If $x + \frac{1}{x} = 7$, then the value of $x^6 + \frac{1}{x^6}$ is?

यदि $x + \frac{1}{x} = 7$, तो $x^6 + \frac{1}{x^6}$ का मान क्या है?

[A] ~~113682~~ ✗

[B] ~~103682~~ ✓

[C] ~~103882~~ ✗

[D] ~~103862~~ ✗

$$x^3 + \frac{1}{x^3} = 7^3 - 3 \times 7$$

$$= 322$$

$$322^2 - 2$$

DS

$$4 - 2 = 2$$

9. If $\left(a + \frac{1}{a}\right) = 7\sqrt{3}$, then what is the value of $(a^6 + a^{-6})$?

यदि $\left(a + \frac{1}{a}\right) = 7\sqrt{3}$ है, तो $a^6 + a^{-6}$ का मान ज्ञात कीजिए।

☒ [A] 3048190

[B] 3048542

[C] 3048132

[D] 3048625

$$a^2 + \frac{1}{a^2} = (7\sqrt{3})^2 - 2$$

$$= 145$$

$$a^6 + \frac{1}{a^6} = 145^3 - 3 \times 145$$

$$= \dots 5 - \dots 5$$

10. If $\left(x - \frac{1}{x} = 4\right)$, then what is the value of $\left(x^6 + \frac{1}{x^6}\right)$?

यदि $\left(x - \frac{1}{x} = 4\right)$ है, तो $\left(x^6 + \frac{1}{x^6}\right)$ का मान क्या है?

SSC CHSL TIER – I 2022

[A] 4689

[B] 4786

[C] 5832

[D] 5778

$\rightarrow 76^2 + 2$

$$\begin{aligned} x^3 - \frac{1}{x^3} &= 4^3 + 3 \times 4 \\ &= 76 \end{aligned}$$

CONCEPT



यदि $\left(x + \frac{1}{x}\right) = 4 \Rightarrow x - \frac{1}{x} = \sqrt{4^2 - 4} = \sqrt{12}$

$$x^6 - \frac{1}{x^6} = \left(x^3 + \frac{1}{x^3}\right) \cdot \left(x^3 - \frac{1}{x^3}\right)$$

$$\downarrow \qquad \qquad \downarrow$$
$$5^2 \times 15\sqrt{12}$$

$$= 780 \times 2\sqrt{3} = 1560\sqrt{3}$$

11. If $\left(x + \frac{1}{x}\right) = 5\sqrt{2}$ & $x > 1$, what is the value of $\left(x^6 - \frac{1}{x^6}\right)$?

यदि $\left(x + \frac{1}{x}\right) = 5\sqrt{2}$ & $x > 1$, तो $\left(x^6 - \frac{1}{x^6}\right)$ का मान क्या है?

[A] $22970\sqrt{23}$

[C] $23060\sqrt{23}$

[B] $23030\sqrt{23}$

[D] $22960\sqrt{23}$

$$(5\sqrt{2})^3 - 3 \times 5\sqrt{2}$$

$$x - \frac{1}{x} = \sqrt{(5\sqrt{2})^2 - 4}$$

$$= \sqrt{46}$$

$$235 \times (100 - 2)$$

$$\left(x^3 + \frac{1}{x^3}\right) \cdot \left(x^3 - \frac{1}{x^3}\right)$$

$$\downarrow \quad \downarrow$$

$$235\sqrt{2} \times 49\sqrt{46}$$

$$= 235 \times 49 \times 2\sqrt{23}$$

$$= 23030\sqrt{23}$$

IF,

$$x + \frac{1}{x} = 4$$



CONCEPT



तो $x^7 \oplus \frac{1}{x^7} = \left(x^4 + \frac{1}{x^4}\right) \left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)$

$\downarrow \quad \downarrow \quad \downarrow$
 $194 \times 52 - 4$

$$x^2 + \frac{1}{x^2} = 14$$

$$x^4 + \frac{1}{x^4} = 194$$

$$= 10084$$

$$194 \times (50 + 2) = 9700 + 388$$

$$x^7 \rightarrow x^4 \times x^3$$

12. If $x > 0$ & $x^4 + \frac{1}{x^4} = 142$, what is the value of $x^7 + \frac{1}{x^7}$?

यदि $x > 0$ & $x^4 + \frac{1}{x^4} = 142$, तो $x^7 + \frac{1}{x^7}$ का मान क्या है?

[A] $1561\sqrt{14}$

[C] $1563\sqrt{14}$

[B] $1560\sqrt{14}$

[D] $1562\sqrt{14}$

$$x^2 + \frac{1}{x^2} = \sqrt{144} = 12$$

$$x + \frac{1}{x} = \sqrt{14}$$

$$\left(x^4 + \frac{1}{x^4}\right) \cdot \left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)$$

$$142 \times 11\sqrt{14} - 1\sqrt{14}$$

$$= 1562\sqrt{14} - \sqrt{14}$$

13. If $x > 0$ & $x^4 + \frac{1}{x^4} = 2207$, then find $x^7 + \frac{1}{x^7}$?

यदि $x > 0$ & $x^4 + \frac{1}{x^4} = 2207$, तो $x^7 + \frac{1}{x^7}$ का मान क्या है?

[A] 710649

[B] 710647

[C] 710654

[D] 710661

$$\sqrt{2209} = 47$$

$$x + \frac{1}{x} = \sqrt{49} = 7$$

$$2207 \times 3 - 7 = ?$$

IF, $x - \frac{1}{x} = 3$



CONCEPT



तो $x^7 - \frac{1}{x^7} = \left(x^4 + \frac{1}{x^4}\right)\left(x^3 - \frac{1}{x^3}\right) + \left(x - \frac{1}{x}\right)$

if, $x \pm \frac{1}{x} = k$

तो $x^7 - \frac{1}{x^7} = ?$

$$\left(x^4 + \frac{1}{x^4}\right) \cdot \left(x^3 - \frac{1}{x^3}\right) = x^7 - x + \frac{1}{x} - \frac{1}{x^7} \quad + \times -$$

$$\left(x^4 + \frac{1}{x^4}\right) \cdot \left(x^3 - \frac{1}{x^3}\right) + \left(x - \frac{1}{x}\right) = x^7 - \frac{1}{x^7}$$

14. If $x^5 - \frac{1}{x^5} = \sqrt{7}$, then find $x^{35} - \frac{1}{x^{35}} = ?$

यदि $x^5 - \frac{1}{x^5} = \sqrt{7}$ है, तो $x^{35} - \frac{1}{x^{35}}$ का मान ज्ञात कीजिए?

[A] $791\sqrt{7}$

[B] $789\sqrt{7}$

[C] $790\sqrt{7}$

[D] $792\sqrt{7}$

$$x^2 + \frac{1}{x^2} = \sqrt{7}^2 + 2 = 9$$

$$x^4 + \frac{1}{x^4} = 9^2 - 2 = 79$$

$$\left(x^4 + \frac{1}{x^4}\right) \cdot \left(x^3 - \frac{1}{x^3}\right) + \left(x - \frac{1}{x}\right)$$

$$\downarrow \quad \downarrow$$
$$79 \times 10\sqrt{7} + \sqrt{7}$$
$$= 791\sqrt{7}$$

15. If $x + \frac{1}{x} = 4$, then, $x^7 - \frac{1}{x^7} = ?$

अर्थात्, यदि $x + \frac{1}{x} = 4$ है, तो, $x^7 - \frac{1}{x^7} = ?$

[A] $5844\sqrt{3}$

[C] $5824\sqrt{3}$

[B] $5842\sqrt{3}$

[D] $5822\sqrt{3}$

$$x - \frac{1}{x} = \sqrt{4^2 - 4} = \sqrt{12}$$

$$\left(x^4 + \frac{1}{x^4}\right) \cdot \left(x^3 - \frac{1}{x^3}\right) + \left(x - \frac{1}{x}\right)$$

$$194 \times 15\sqrt{12} + \sqrt{12}$$

$$= 2911\sqrt{12} = 2911 \times 2\sqrt{3}$$

#

16. If $x + \frac{1}{x} = 3$, then, $x^8 + \frac{1}{x^8} = ?$

यदि $x + \frac{1}{x} = 3$ है, तो $x^8 \oplus \frac{1}{x^8} = ?$

[A] 2425

[C] 2329

[B] 2207

[D] 3234

$$x^2 + \frac{1}{x^2} = 3^2 - 2 = 7$$

$$x^4 + \frac{1}{x^4} = 7^2 - 2 = 47$$

$$x^8 + \frac{1}{x^8} = 47^2 - 2 = 2207$$

17. If $\left(x - \frac{1}{x}\right) = \sqrt{6}$ & $x > 1$, then what is the value of $\left(x^8 - \frac{1}{x^8}\right)$?

यदि $\left(x - \frac{1}{x}\right) = \sqrt{6}$ & $x > 1$, तो $\left(x^8 - \frac{1}{x^8}\right)$ का मान क्या है?

[A] ~~1024~~ $\sqrt{15}$

[B] 992 $\sqrt{15}$

[C] ~~998~~ $\sqrt{15}$

[D] ~~1012~~ $\sqrt{15}$

$$x^8 - \frac{1}{x^8} = \left(x^4 + \frac{1}{x^4}\right) \cdot \left(x^2 + \frac{1}{x^2}\right) \cdot \left(x + \frac{1}{x}\right) \cdot \left(x - \frac{1}{x}\right)$$

$\downarrow \quad \quad \downarrow \quad \quad \downarrow \quad \quad \downarrow$
 $6x \quad \quad 8x \quad \quad \sqrt{10} \quad \quad \sqrt{6}$

$$= 496x^2\sqrt{15} = 992\sqrt{15}$$